Hypothetical learning trajectory design on the history of Indonesian independence struggle in Mathematics logic instruction

Dra Nurjanah
Nusantara Islamic University, Indonesia
Email: nurjannah@ini-itu.info

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ABSTRACT

The research was aimed at developing learning resources for mathematics logic using the hypothetical learning trajectory designed through reflection on the history of Indonesian independence struggle. The study was carried out at the Department of Mathematics Education of the Faculty of Teacher Training and Educational Sciences Nusantara Islamic University (Uninus). The study used design research consisted of three stages: preparing the experiments, design the experiments, and retrospective analysis. Preparing the experiments stage has been completed and design the experiments stage is currently under preparation. The main activities accomplished in preparing the experiments stage consisted of: studies of Indonesian independence struggle, curriculum analysis, literature review, and early prototype design. Design the experiments phase has enabled the development of the research instruments. The learning trajectory which has been designed in the first stage involved: reflections on the history of Indonesian independence struggle; implication and bi-implication; implication, bi-implication and their truth in the context of the history of the Indonesian independence struggle; and implication, bi-implication and their truth in the context of mathematics. Based on the results of discussions with colleagues, the students' ability in mathematical thinking can be developed by using the history of Indonesian independence struggle as the context of learning in a mathematics logics course.
Keywords: *History, Indonesia, Independence, Learning, Trajectory, Mathematics Logic.*

INTRODUCTION

Students' inability to connect theories they learn in school and realities they find in real-life situations is a very common problem in education. This condition is the result of poor interconnectedness among materials in the subjects taught in schools. As the consequence, students perceive them as less meaningful learning experiences. Especially in mathematics instruction, it reinforced the notion that this subject is a heavy category, and as an effect many learners dislike to study mathematics.

To deal with this issue, a contextual instruction can become an alternative solution to help students to see the meaningfulness of materials they learn through connecting them with daily real-life situations, either in their personal life or in their socio cultural environment. As Sauaian (2002) say that "Contextual-based teaching is teaching through focusing on selected topics and resorting to environmental orientations" (p. 17). In more details, Berns & Erickson (2001) identify that "Contextual teaching and learning is a conception of teaching and learning that helps teachers relate subject matter content to real world situations; and motivates students to make connections between knowledge and its applications to their lives as family members, citizens, and workers and engage in the hard work that learning requires" (p. 2).

Contextual instruction can only be implemented successfully if both students and teachers aware of the realization of the learning objectives. If Sauian, Berns and Erickson emphasize heavily on the important role that the teachers can play in the initial steps of learning process, Johnson (2002) sought to view the model from the perspective of students. As defined by Johnson, contextual teaching and learning is "a system of instruction based on the philosophy that students learn when they see meaning in academic material, and they see meaning in schoolwork when they can connect new information with prior knowledge and their own experience".

Teachers may guide students to connect the materials they learn in classroom and real-life situation through many ways; one of which is through developing learning materials that has connection with students' actual experiences and providing more practical sessions. For example, the history of Indonesian independence struggle (HIIS) can be used as the context of mathematics logic instruction; and the celebration of independence day can be used as the context of: integers, sets, relations, and functions instruction. The formulized problems in the study are as follows: 1) how should the hypothetical learning trajectory (HLT) be designed using the context of HIIS; 2) how are students' mathematical reasoning when the HIIS used as the context of learning in a mathematics logics course?

METHODOLOGY

This study employed design research method to find answers to the questions and reach the objectives. According to Akker (2006), design research is consist of: 1) preparing the experiments, 2) design the experiments, and 3) conducting retrospective analysis. The following sections will elaborate the procedures of the study. This study involved instructors and first year students in the Department of Mathematics Education, Faculty of Education, Nusantara Islamic University in the academic year 2015/2016.

The stage of preparing the experiment is consist of two parts: preliminary design and pilot experiment. In the preliminary design, this study begun with reviewing literatures or references analysis, curriculum analysis, and designs the HLT. The center of attraction of references analysis are: contextual instruction, HIIS, and mathematics logic. Curriculum analysis focused on mathematics logic's syllabus and course plan. In reference to literature review and analysis of curriculum, this study decided to use the HIIS as the learning context of implication and bi- implication.

Clements & Sarama (2004) stated that a "hypothetical learning trajectory included the learning goal, the learning activities, and the thinking and learning in which the students might engage" (p. 82). In designing the HLT, this study developed hunches about strategy, thinking, and reasoning patterns that the students developed from informal to formal state. These hunches were developed to anticipate students' reasoning strategies that rose and developed throughout the learning process. The hunches of the learning trajectory are dynamic and always keep updated in accordance with students' behavior throughout the research process. Meanwhile, pilot experiment stage was aimed at bridging the stage of preliminary design and design experiment. The purpose of pilot experiment was to study about students' initial abilities and their adjustment towards the learning process using HLT designed by instructor. In this study, pilot experiment was conducted through trying the learning to ten students at lower, middle, and upper class.

In the stage of design experiment, the study conducted learning activities using HLT that has been developed in the stage of preparing for the experiment, especially in the preliminary design stage. In this stage, this study collected data about learning process in classroom and students' reasoning process based on the perspectives of social, mathematical practices, psychological practices, and the concept and activities of mathematics.

Meanwhile, Retrospective analysis was intended to analyze the collected data from the stage of design experiment. The data included video recorded of learning process, audio recorded of interview with the students and instructors, students' worksheet, and field-note during the observation. In the retrospective analysis stage, reconstructions and revision if HLT were conducted in order to respond to the implications of theories and the implementation of the HLT in broader context and settings.

RESULTS AND DISCUSSION

This study has completed the stage of preparing for the experiment and in the process of preparing the stage of design the experiment. The main activities conducted in this stage included: reflections on the HIIS, analysis of curriculum especially those parts related to the syllabus of Set Theory and Logic, literature review, design the HLT; and pilot experiment. The stage of HLT design was intended to develop learning trajectory that the students will go through in the learning activities about conditional and biconditional. In general, HLT that has been designed to deliver the implication and bi-implication materials can be seen in the Figure 1.

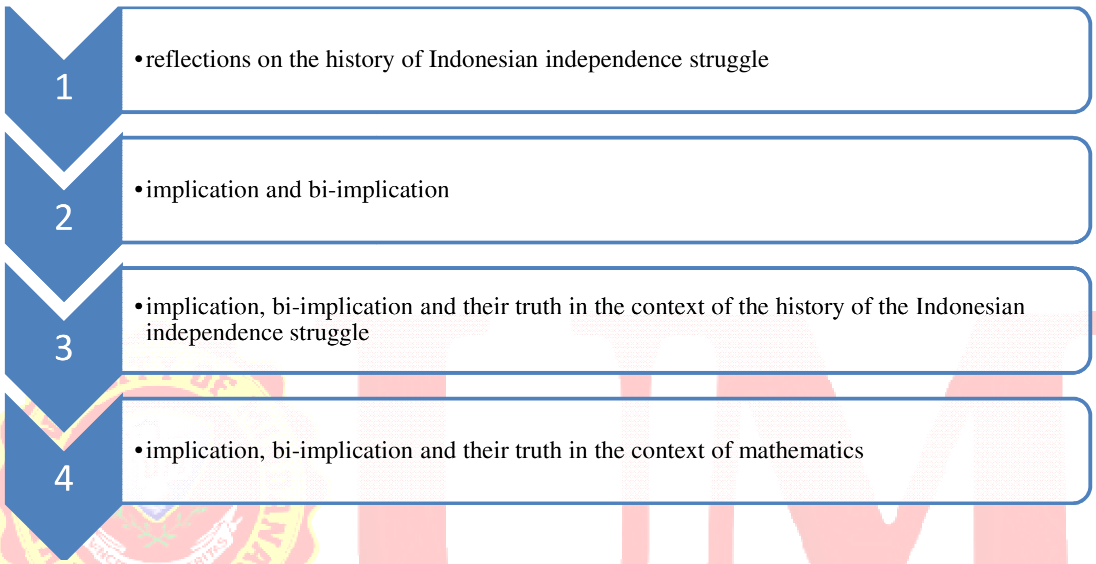

Figure 1 Learning Trajectory for implication and bi-implication.

Reflections on the HIIS

These activities were designed to provide the students with the learning contexts of mathematics logic and help them understand conditional and biconditional more easily. In the beginning of the learning activities, the instructor asked the students to express statements about the HIIS. The statements expressed by the students were starting points for developing conditional and biconditional statements. It is in accordance with Teppo (2003) who says that, “it has long been recognized that humans have an understanding of conditionals in a social context that may not be closely related to their understanding of conditionals in a more-abstract context such as mathematics.” (p.4)

Implication and Bi-implication

The form of implication (conditional) is “if A then B”, symbolized by $A \rightarrow B$. The conditional statement $A \rightarrow B$ is frequently named a deduction, and you can state that A indicates B. Any other method of expressing $A \rightarrow B$ is to state that A is an adequate circumstances for B, that is the right or wrong of A ensures the right or wrong of B. In other words, B is an essential circumstances for A, while B essentially come after A. The single statement A is named the hypothesis or antecedent, and B is named the conclusion or consequent. The compound statement $(A \rightarrow B) \wedge (B \rightarrow A)$ is a conjunction of two conditional statements. When we combine two conditional statements this way, we have a bi-conditional or bi-implication.

Implication and its truth value in the context of HIIS

In this stage, the instructor asked the students to develop some implication statements in the context of HIIS. Then, the instructor asked the students to focus on the following statement, “If Tjut Nyak Dhien is a warrior of Indonesian war of Independence from Aceh; then to weaken her influence on her followers, Tjut Nyak Dhien was exiled out of Aceh". By appointment: $\tau(A)$ = the truth value of A, T = True, F = False; the instructor asked the students to analyze four possible implications and identify the truth value for each implication as shown in the Table 1.

Table 1. Examples of conditional statements in the context of HIIS.

A	B	$\tau\ (A \rightarrow B)$
Tjut Nyak Dhien is a warrior of Indonesian war of independence from Aceh $\tau(A) = $ T	To weaken her influence on her followers, Tjut Nyak Dhien was exiled out of Aceh $\tau(B) = $ T	T
Tjut Nyak Dhien is a warrior of Indonesian war of independence from Aceh $\tau(A) = $ T	To weaken her influence on her followers, Tjut Nyak Dhien was not exiled out of Aceh. $\tau(B) = $ F	F
Tjut Nyak Dhien is not a warrior of Indonesian war of independence from Aceh $\tau(A) = $ F	To weaken her influence on her followers, Tjut Nyak Dhien was exiled out of Aceh $\tau(B) = $ T	T
Tjut Nyak Dhien is not a warrior of Indonesian war of independence from Aceh $\tau(A) = $ F	To weaken her influence on her followers, Tjut Nyak Dhien was not exiled out of Aceh $\tau(B) = $ F	T

After studying the examples above, the students are expected to easily understand that an implication will only wrong if the antecedent are correct but the conclusion is wrong. According to Bloch (2011), implication between A and B, symbolized by $A \rightarrow B$, can be understood at a glance that is always right for all possibility, exept A is correct and B is wrong. Then, the instructor asked the students to draw the implication truth value given in Table 2.

Table 2. Implication truth table

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Then, the instructor explained that in an absolutely reasonable feel, implication statement $A \rightarrow B$ essentially don't indicate an action and reaction between the parts A and B, even though in mathematics and in common discussion they happen. By a perspective logic, the statement, "if Soekarno was being exiled to the Island of Buru, then the national flag was sewed by Mrs. Fatmawati" is a legitimate conditional, even though the fact that Soekarno was actually being exiled to the Island of Buru was not the cause of the sewing of the Flag by Mrs. Fatmawati. The conditional is correct since the statement of Soekarno was being exiled to the Island of Buru is a correct statement, and the flag was sewed by Mrs.Fatmawati is a correct statement too. Once again, even though we can not found correlation between the component parts (antecedent and consequent).

Implication and its truth value in the context of Mathematics

After studying the examples of implication statements in the context of HIIS, the students are expected not to find difficulties to understand the implications in mathematics contexts. The discussion about truth value table also intended to help the students accept any implications whether the hypotheses and conclusions are interrelated or not interrelated. In addition, the instructor explained that hypotheses and conclusion of an implication could be drawn not in the form of a statement (since the truth values still cannot be determined yet). This kind of conditional truth value can be found in the relation between the two components. Examples of conditional statements in mathematics contexts and mixed contexts as well as their hypotheses and conclusions conditions are shown in the Table 3.

Table 3. Samples of conditional in Mathematics context

Hypothesis-Conclusion	$A \rightarrow B$	Truth Value
Unrelated	$2 + 6 = 8 \rightarrow 7 \times 3 = 21$	T
Related	3 is prime $\rightarrow$ 3^2 is prime	F
Not a statement	$2x + 5 = 17$, then $x = 12$	F
Not a statement	If m is an odd number between 5 and 25, then m is counting number	T
Unrelated	If General Soedirman were the proclamator, then 2+3 =5	T

Biconditional and truth value in the context of HIIS

In this stage, instructor asked the students to make some bi-implication statements in the context of HIIS. Then, instructor asked the students to focus on the following statement, "Japan surrendered to allies on August 15th, 1945 if and only if the independence of Indonesia was proclaimed on August 17th, 1945". Next, the instructor asked the students to consider four possible bi-implications formed and identify the truth value for each bi-implications, as shown in Table 4.

Table 4. Examples of Bi-implication statements in the context of HIIS

A	B	$\tau(A \leftrightarrow B)$
Japan surrendered to allies on August 15th, 1945 $\tau(A)$= T	The independence of Indonesia was proclaimed on August 17th, 1945 $\tau(B) =$ T	T
Japan surrendered to allies on August 15th, 1945 $\tau(A)$= T	The independence of Indonesia was not proclaimed on August 17th, 1945 $\tau(B) =$ F	F
Japan did not surrender to allies on August 15th, 1945 $\tau(A)$= F	The independence of Indonesia was proclaimed on August 17th, 1945 $\tau(B) =$ T	F
Japan did not surrender to allies on August 15th, 1945 $\tau(A)$= F	The independence of Indonesia was not proclaimed on August 17th, 1945 $\tau(B) =$ F	T

After studying the example above, the students were expected to understand that a bi-implication will be correct ones if the antecedent and consequence have the same truth value. Then, the instructor asked one of the students to write the truth value table of bi-implications on the board as shown in Table 5:

Table 5. Bi-implication Truth Table

A	B	$A \leftrightarrow B$
T	T	T
T	F	F
F	T	F
F	F	T

Bi-implication and it's truth value in the context of Mathematics

After the students have understood the examples of bi-implication in the context of HIIS, they are expected to understand bi-implications in the context of mathematics more easily. In addition hypotheses-conclusion condition that have been previously explained, the instructor also asked the students to pay attention to specific definitions of mathematics concepts in the form of biconditional statements. Theorems of "A $\leftrightarrow$ B" are deeply precious in mathematics and it provides similar and attractive other method to express correctly the equivalent information. We may recognized statements of this category in Euclidean geometry. The formula: A polygon is a triangle $\leftrightarrow$ it has exactly 3 sides, indicate that triangle same with polygon that has exactly 3 sides , or in the other word "triangle" defined by "polygon that has exactly 3 sides." Examples of biconditional statements in the mathematics contexts and the mixed contexts as well as hypotheses and conclusion conditions, are presented in Table 6.

Table 6. Examples of bi-implication in the context of Mathematics

Hypothesis-Conclusion	**Biconditional**	**Truth Value**
Unrelated	$2 + 6 = 8 \leftrightarrow 7 \times 3 = 21$	T
Related	0.25 is decimal fraction ↔ 4 x 0.2 5 is decimal fraction	F
Not a statement	The painting was beautiful ↔ a counting number is divided by 5	F
definition	Someone got the title of *Hajji Mabrur* if and only if his behaviour after hajj better then ever	T
Not a statement	$x + 2 = 7 \leftrightarrow x = 5$	T
definition	A triangle is isosceles if and only if it has two congruent (equal) sides.	T
Unrelated	Berlian is a hero if and only if 7^2 is odd number	F

RESULTS AND DISCUSSION

Based on the observation in the pilot experiment stage, the study found that the HIIS can be used as the learning context of implication and bi-implication. The prominent figures the students identified when they were asked to make statements about the HIIS included: General Soedirman, Soekarno, Muhammad Hatta, Pangeran Diponegoro, Pattimura, and Tjut Nyak Dhien. Meanwhile, the most commonly identified event in the history of Indonesian independence struggle included: the war of Aceh, the war of Diponegoro, the kidnapping of Bung Karno by youth leaders to force him to proclaim the independence, and the proclamation of independence. The history had become a means for the students to develop the table of truth and understand the truths of implication and bi-implication.

In the next step, the instructor facilitated the students to convert their understanding about conditional and biconditional based on the context of HIIS into the context of mathematics. According to Freudenthal (2002) "Mathematics has arisen and arises through mathematising. This phenomenological fact is didactically accounted for by the principle of guided reinvention. Mathematising is mathematising something─something non-mathematical or something not yet mathematical enough, which needs more, better, more refined, more perspicuous mathematising." (p. 66). Then, the study illustrated the experiment of HLT in mathematics context where there were exciting classroom interactions. In the learning process, all students were highly engaged in discussions.

This mathematising process begun by presenting the following statement: "$2 + 6 = 8 \rightarrow 7 \times 3 = 21$," which led to dialog and interaction; all names of students are anonymous.

Nendi : Mam, what is the relation between 2 + 6 = 8 and 7 x 3 = 21?
Instructor :Anyone can help answer Nendi's question?
Firman :No relation, Mam!
Berlian : I believe there's a relation, Mam!
Instructor :What is it, Berlian?
Berlian : Both are correct
Instructor : Firman was right. The product of 7 and 3 cannot be determined by the sum of 2 and 6. No matter how many the sum of 2 and 6, 7 x 3 fixed = 21.
Firman : Yes, Mam!
Instructor : Since 2 + 6 = 8 is right, and 7 x 3 = 21 also right, as Berlian said, then $2 + 6 = 8 \rightarrow 7 \times 3 = 21$ is a

Correct conditional statement.

Firman&
Berlian : Yes, Mam!
Instructor : What if General Soedirman were a proclamator, then 2+3 = 7?
Tanti : I think there's no relation on this at all, Mam!
Ninda : Not only no relation, but also both are wrong.
Berlian : But Mam, General Soedirman was a proclamator is a statement with wrong value, 2+3 = 7 also has wrong value, then …
Instructor : So, Berlian, what do you say?
Berlian : So, the statement has a truth value.
Instructor : Exactly, give it up for Berlian you folks!

Based on this mathematising process, the students did not get any significant difficulties to understand or accept the statement "2 + 6 = 8 ↔ 7 x 3 = 21" as a truth statement.Then, in order to facilitate the students to do mathematical process on biconditional statement, the instructor expressed a statement: x+2= 7↔ x=5 with the hope that there will be even more exciting conversation in class since the antecedent and consequence of that biconditional was not a statement. Since this activity is important, the instructor then redesigned the learning process to make the following conversation:

Instructor : Let's focus on this statement: $x + 2 = 7 \leftrightarrow x = 5$. Do you find anything special in this bi-conditional?
Since no one responded, the instructor then changed the strategy.
Instructor : Berlian, can you name the antecedent and consequence of this biconditional!
Berlian : Antecedent: $x + 2 = 7$, consequence: $x = 5$
Instructor : Are the antecedent and consequence a statement?
Students : Not, Mam!
Ani : But, Mam, if $x + 2 = 7$, then x exactly $= 5$, meaning that …
Instructor : Meaning what, Ani?
Ani : The biconditional is correct.
Instructor : Give it up for Ani, folks!
Remember the rule. If antecedent and consequence of a conditional or biconditional are not a statement, then the right or wrong of the statement cannot be decided by the right or wrong of antecedent and consequence, rather by the correlation of both.

CONCLUSION

The students' ability in mathematical reasoning can be developed by using the history of Indonesian independence struggle as the context of learning in a mathematics logics course.

REFERENCES

Akker, J., van den. (2006). *Educational Design Research*. Routledge. London and New York.

Berns, R.G. and Erickson, P.M. "Contextual Teaching and Learning: Preparing Students for the New Economy", The Highlight Zone: Research @ Work No. 5,2001. Retrieved October 27, 2013 from:http://www.cord.org/uploadedfiles/NCCTE_Highlight05-ContextualTeachingLearning.pdf.

Bloch, E.D. (2015). *Proofs and Fundamentals A First Course in Abstract Mathematics*. 2nd Edition. New York Dordrecht Heidelberg London: Springer

Clements, D.H., Sarama, J. (2004). Learning Trajectories in Mathematics Education. In *International Journal of Mathematical Thinking and Learning, 6,* (2), 81-89.

Freudenthal, H. (2002). *Revisiting Mathematics Education*. Kluwer Academic Publishers New York, Boston, Dordrecht, London, Moscow

Johnson, E. B. (2002). "Contextual Teaching and Learning: What It is and Why It's Here to Stay. California: Corwin Press.

Sauian, M.S. . (2002). *Mathematics education: The relevance of "contextual teaching" in developing countries.* In P. Valero & O. Skovsmose (2002) (Eds.). Proceedings of the 3rd International MES Conference. Copenhagen: Centre for Research in Learning Mathematics, pp.

Teppo. A.R., et al. (2003). *The Assessment Of Mathematical Logic: Abstract Patterns and Familiar Contexts*. Proceedings of the 27th International Group for the Psychology of Mathematics Education Conference Held Jointly with the 25th PME-NA Conference (Honolulu, Hawaii, July 13-18, 2003). Volume 4. Retrieved February 25, 2015 from: http://files.eric.ed.gov/fulltext/ED501135.pdf.